Development of a multi-scale process-based eco-physiological model

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Aims

- process-based model
- ecological and population genetic dynamics of traits
- antagonistic eco-physiological interactions
- linking within- and between-host dynamics
- beetles $+$ trees

Meshing pre-existing models

Tree population dynamics

Berryman, Stenseth, and Wollkind. 1984. Metastability of forest ecosystems infested by bark beetles. *Res. Pop. Ecol.* 26:13-29.

Dispersal and aggregation process

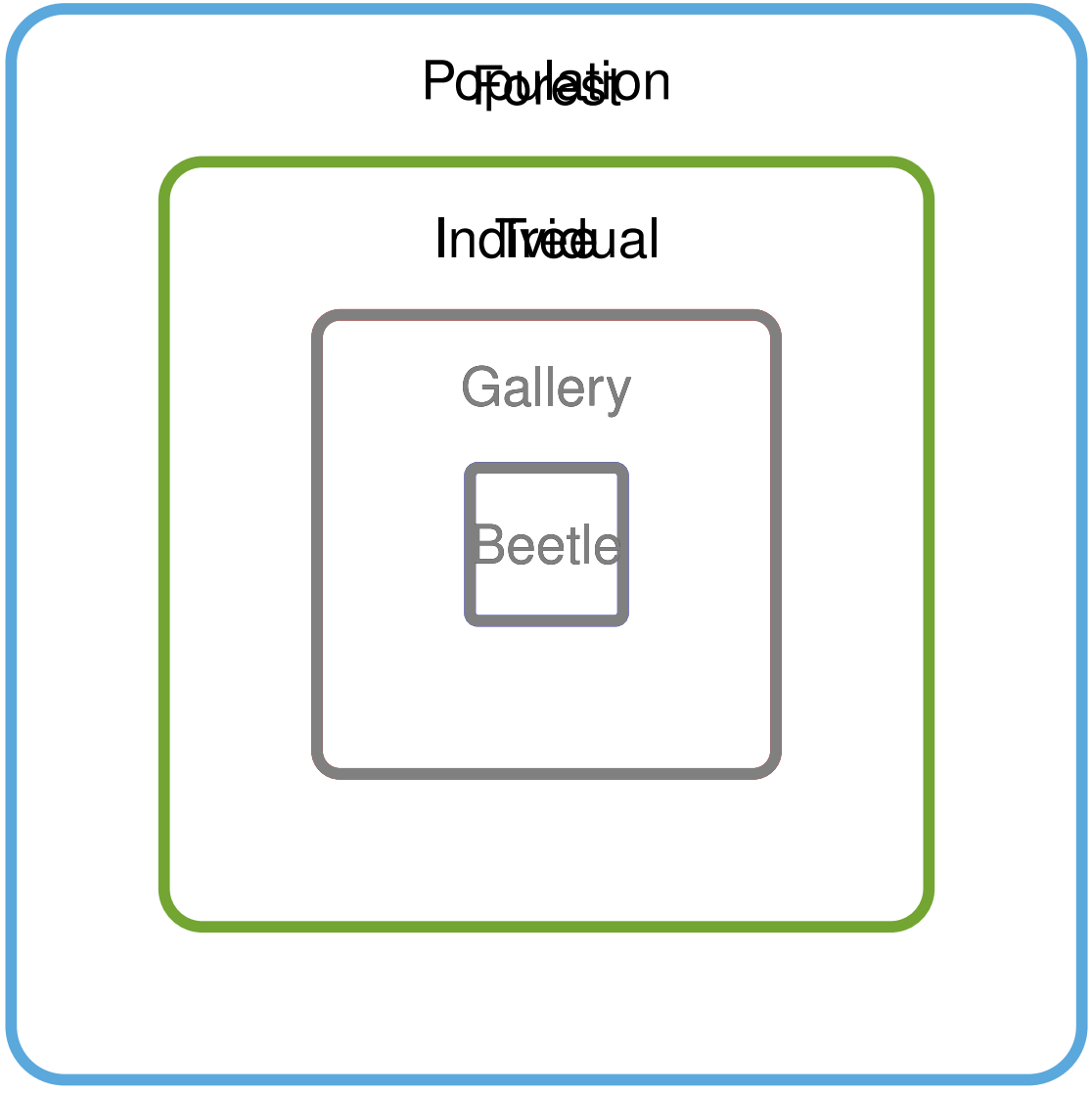
Berryman, Raffa, Millstein, and Stenseth. 1989. Interaction dynamics of bark beetle aggregation and conifer defense rates. *Oikos* 56:256–263.

Within-tree physiological interactions

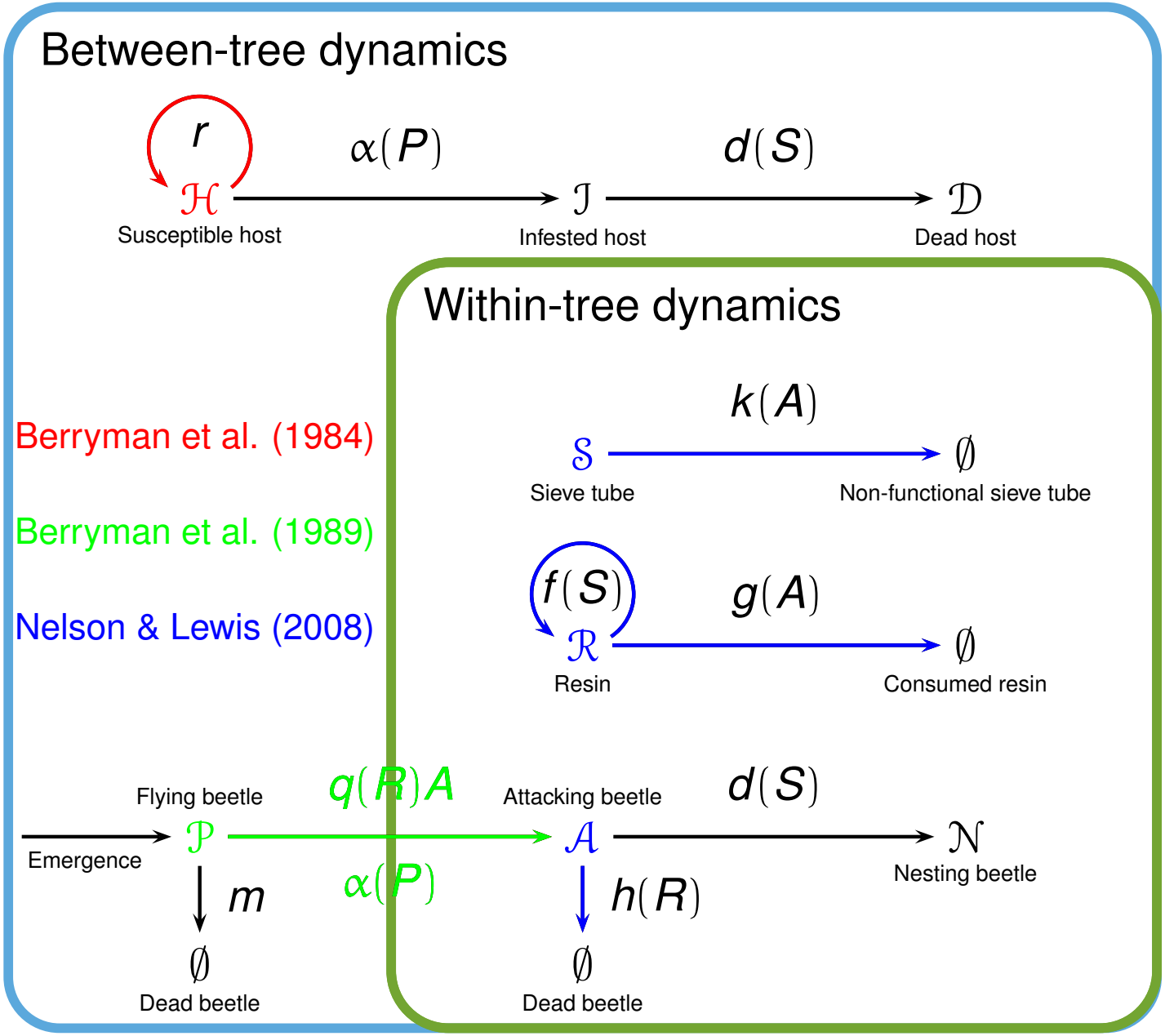
Nelson and Lewis. 2008. Connecting host physiology to host resistance in conifer-bark beetle systems. *Theoretical Ecology* 1:163–177.

Multi-scale biological processes

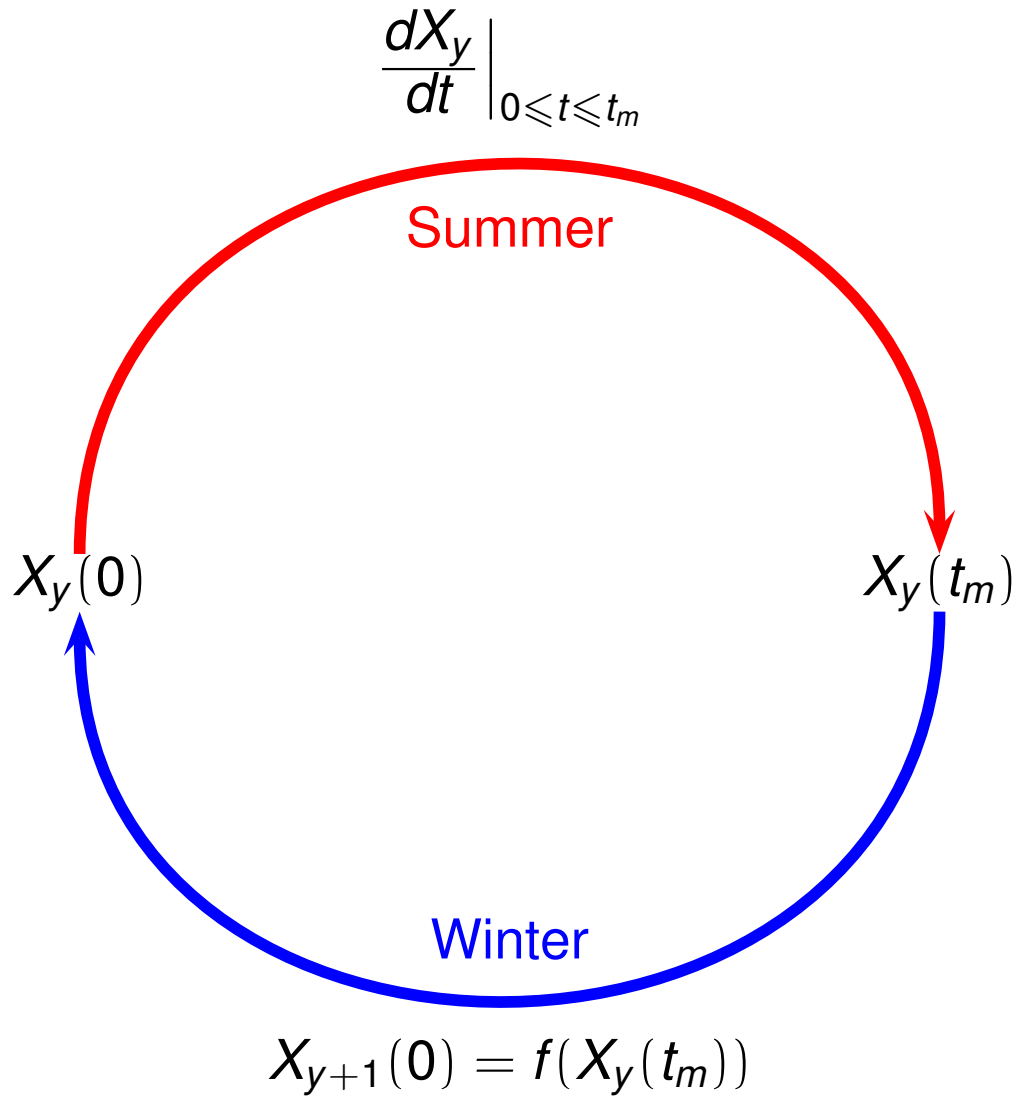
Nested eco-physiological model



Pre-existing models



Within and between season dynamics



The model

Summer

$$
\begin{aligned}
\frac{dH_y}{dt} &= rH_y - \alpha(P_y)H_y \\
\frac{dI_y}{dt} &= \alpha(P_y)H_y - d(S_y)I_y \\
\frac{dD_y}{dt} &= d(S_y)I_y \\
\frac{dS_y}{dt} &= -k(A_y)S_y \\
\frac{dR_y}{dt} &= f(S_y)R_y - g(A_y)R_y \\
\frac{dP_y}{dt} &= -I_y(P_y(q(R_y)A_y)) - \alpha(P_y)H_y - mP_y \\
\frac{dA_y}{dt} &= P_y(q(R_y)A_y) + \alpha(P_y) - h(R_y)A_y - d(S_y)A_y \\
\frac{dN_y}{dt} &= d(S_y)A_y
\end{aligned}
$$

Winter

$$
\begin{aligned}
H_{y+1}(0) &= H_y(t_m) \\
P_{y+1}(0) &= H_y\left(A_y(t_m)\exp\left(1 - \frac{A_y(t_m)}{K(S_y(t_m))}\right)\right) \\
A_{y+1}(0) &= 0 \\
S_{y+1}(0) &= S_y(t_m) \\
R_{y+1}(0) &= R_y(t_m)
\end{aligned}
$$

Future directions

Genetic component...

Aim: Define and formalize the quantitative genetic basis for intra- and interspecific antagonistic eco-physiological interactions

Spatial dynamics...

Aim: Spatially explicit model at the landscape level

Ecological interactions...

Aim: Incorporate antagonistic physiological interactions between fungi population and tree

Thank you for your attention

Image: MPK

...and if all else fails, resort to good ol' elbow grease.
Mountain pine beetles - eat this!

Full continuous time model

Do not worry about your difficulties in Mathematics. I can assure you mine are still greater.

$$
\begin{aligned}
\frac{dH_y}{dt} &= r_0 H_y \left(1 - \frac{H_y}{K}\right) - \alpha_0 H_y P_y \\
\frac{dI_y}{dt} &= \alpha_0 H_y P_y - d_0 \left(1 - \left(\frac{S_y}{S_0}\right)^c\right)^{(1/c)} I_y \\
\frac{dD_y}{dt} &= d_0 \left(1 - \left(\frac{S_y}{S_0}\right)^c\right)^{(1/c)} I_y \\
\frac{dP_y}{dt} &= -I_y (P_y ((uR_m R_y - vR_y^2) A_y)) - \alpha_0 H_y P_y - m_0 P_y \\
\frac{dA_y}{dt} &= P_y ((uR_m R_y - vR_y^2) A_y) + \alpha_0 P_y - h_0 A_y R_y - d_0 \left(1 - \left(\frac{S_y}{S_0}\right)^c\right)^{(1/c)} A_y \\
\frac{dN_y}{dt} &= d_0 \left(1 - \left(\frac{S_y}{S_0}\right)^c\right)^{(1/c)} A_y \\
\frac{dS_y}{dt} &= -k_0 S_y A_y \\
\frac{dR_y}{dt} &= f_0 \left(1 - \frac{R_y}{R_m}\right) S_y - g_0 A_y R_y
\end{aligned}
$$

Number of susceptible trees

$$\frac{dH_y}{dt} = r_0 H_y \left(1 - \frac{H_y}{K}\right) - \alpha_0 H_y P_y$$

Number of infested trees

$$\frac{dI_y}{dt} = \alpha_0 H_y P_y - d_0 \left(1 - \left(\frac{S_y}{S_0}\right)^c\right)^{(1/c)} I_y$$

Number of beetle killed trees

$$\frac{dD_y}{dt} = d_0 \left(1 - \left(\frac{S_y}{S_0}\right)^c\right)^{(1/c)} I_y$$

Number of dispersing beetles

$$\frac{dP_y}{dt} = -I_y(P_y((uR_mR_y - vR_y^2)A_y)) - \alpha_0 H_y P_y - m_0 P_y$$

Number of attacking beetles

$$\frac{dA_y}{dt} = P_y((uR_mR_y - vR_y^2)A_y) + \alpha_0 P_y - h_0 A_y R_y - d_0 \left(1 - \left(\frac{S_y}{S_0}\right)^c\right)^{(1/c)} A_y$$

Number nesting beetles

$$\frac{dN_y}{dt} = d_0 \left(1 - \left(\frac{S_y}{S_0}\right)^c\right)^{(1/c)} A_y$$

Number sieve tubes

$$\frac{dS_y}{dt} = -k_0 S_y A_y$$

Resin volume

$$\frac{dR_y}{dt} = f_0 \left(1 - \frac{R_y}{R_m}\right) S_y - g_0 A_y R_y$$